ANT COLONY ALGORITHM FOR REPAIRING THE INCONSISTENT MATRIX AHP

Abba Suganda Girsang

Ant Colony Algorithm

for Repairing the Inconsistent Matrix AHP

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PREFACE

First, I am thankful and grateful for my God "Jesus Christ" for His leading me to write this book. Also my professor while I studied in Taiwan, Prof Chu-Sing Yang and Prof. Chun-Wei Tsai for supervising me to conduct the research. The idea of this book is based on that research.

This book proposes using ant colony optimization (ACO), one of good algorithm for optimization, to solve the consistent matrix problem in analytic hierarchy process (AHP). This book discusses ACO in single objective, ANTAHP and ACSICR, and also multi-objective called, MOBAF and MOBAM. These methods are implemented for solving two model matrix, multiplicative preference matrix and fuzzy preference matrix.

Therefore, this book is suitable for a student, lecturer or researcher to learn two topics, modeling ACO and consistency in AHP. It is presented very practical starting modeling, detailing the proposed method (and its modified) for solving inconsistent matrix. In other hand, this book shows that solving inconsistent matrix AHP using metaheuristic method such as ACO, is a fruitful research.

At last, the author would like to thank who help preparing this book. Hopefully, it is useful for knowledge and research.

Jakarta, August 2016

Abba Suganda Girsang

CONTENTS

Chapter 1

Consistent Matrix as an Important Issue on AHP

Analytic hierarchy process (AHP) is a tool on multi criteria decision making (MCDM) to determine the significant criteria and then selecting the best strategy regarding the criteria analysis [1]. In AHP, a comparison matrix is generated to reveal the opinion of decision makers (DMs). Multiplicative preference relations [2, 3] and fuzzy preference relations [4, 5] are two model to state a comparison matrix in AHP.

One of the interesting issues of matrix comparison is the consistency. Without consistent, the comparison maker cannot be used for judgement. Unfortunately, the consistency of matrix comparison is hard to obtain for some reasons. Some reasons make the consistency on multi-criteria decision is hard to obtain especially for large size matrix. The decision maker is difficult to remember his/her opinion to be always consistent. The other problem, the decision maker may lack knowledge or experience to conclude the problem [6].

Usually, there are two approaches to transform inconsistent matrices into consistent ones [6]: (1)The decision makers change the value matrix into new ones

such that till satisfy the acceptable consistency. (2)The value of preference of DMs is modified by the particular method such that the modified one satisfies the acceptable consistency. This issue takes attention for researchers to alleviate the inconsistency of pairwise comparison preference. Saaty firstly defined the inconsistency by proposing the threshold of consistency ratio (CR) as being 0.1[2]. Since then, many researchers gave attention to solve the inconsistent comparison matrix. There are two models comparison matrices, multiplicative preference[7-16] and fuzzy preference[17-24].

In multiplicative preference, as Abba et al mentioned [1] in their research, Siraj et al.[7] improved the consistency by detecting and removing intransitive in the comparison matrices. Dong et al. [8] proposed the method, which has two purposes, to improve the individual consistency measure and to increase the consensus measure. Jose et al. [9]presented a model based on multilayer perceptron (MLP) neural networks to develop missing values and improving its consistency as well. Ishizaka and Lusti [10] proposed an expert module that consists of some parts such as detecting rule transgressions, suggesting alternative and giving hints to continue the comparison process. One of the important steps in this method is to generate the module which has four types: principal diagonal, independent, transitive and

comparisons. Daji et al. [11] modified the inconsistent comparison matrix by proposing the induced matrix to identify the elements, which lead the matrix to be inconsistent. Ultimately, they suggested altering the elements to obtain consistent matrix. Consequently, they did not change most of the elements in the matrix except for the parts suspected of rendering the inconsistent matrix. Jose and Lamata [12] presented an estimation method for a good random coefficient index (RI). They used a simpler function than Saaty to define the accepting or rejecting matrices, and also offered the levels of consistency to consider restrictive situations. Xu and Wei [13] developed a consistent matrix B by replacing the inconsistent matrix A with $b_{ij}=a^{\alpha}_{ij}(W_i/W_j)^{1-\alpha}$, where $W = (W_1 \ldots W_i \ldots W_{ij})^T$ is the eigenvector of A, and α is a positive value close to 1.0. Caoet al. [16] extended Xu and Wei's method by decomposing the original matrix as a Hadamard product of a consistent matrix and a reciprocal deviation matrix. A modified matrix is built via a convex combination of the reciprocal deviation matrix and a zero deviation matrix.

Some researchers tried to solve the problem of inconsistent in a comparison matrix in various approach. In fuzzy preference relations, Xu et al. [17], Xu et al. [18] , Xu et al. [19], and Xu et al. [20] proposed the methods to repair the consistency. Moreover, one of the various issue in AHP is to fulfill the incomplete the fuzzy preference matrix. Liu

et al. [21] and Chen et al. [22] proposed methods to solve the incompleteness preference matrix and also repair the inconsistency in fuzzy preference matrix. Xia et al. [25] repaired the consistency by using the geometric consistency index either in fuzzy preference relation, in both case, complete and incomplete matrices. In reciprocal preference relation, Chicalana et al. [26] proposed a functional equation to model the cardinal consistency in the strength of preferences of reciprocal preference relations [1].

Analytic Hierarchy Process (AHP)

2.1. Model AHP

In solving multi criteria decision making (MCDM), the analytic hierarchy process (AHP) is a decision making tool to obtain a priority alternative [27]. It is designed to cope with both the rational and the intuitive methods to select the best from a number of alternatives evaluated with respect to several criteria. In this process, the decision maker carries out simple pairwise comparison judgments which are then used to develop overall priorities for ranking the alternatives. These comparisons may be taken from actual measurements or from a fundamental scale that reflects the relative strength of preferences and feelings [28]. The detailed procedure of AHP can be summarized in some steps.

Step 1: Defining the decision problems.

The step of decision problem purposes to define the goal, to collect the relevant information and to compose the favorable criteria. Then the decision problem is constructed in form of hierarchy structure which arranges as follows: decision goals, criteria, sub-criteria, and alternatives. The first level indicates the goal for the specific decision

problem, while in the second level, the goal is decomposed to several criteria and the lower levels can be divided into other sub-criteria [1].

Step 2: Generating the comparison matrix.

The comparison matrix is generated by decision maker's opinion to judge the priority of criteria [6]. An element comparison matrix can reflect the subjective opinion that expose strength of the preference and the feeling of the decision maker. Multiplicative preference relations and fuzzy preference relation are two models of comparison matrix. The detailed of two preference relation can be shown on subsection 2.1.1 and 2.1.2.

Step 3: Determining consistent ratio of comparison matrix.

The consistent ratio reflects the consistency of decision making judgment during the evaluation. The comparison matrix cannot be directly used without satisfying the consistency. It is because an inconsistent matrix cannot be used to make decision [6]. Saaty defined the threshold of CR lower than 0.1 [2]. The determining of this CR value can be detailed in Section 2.2.

Step 4. Determining the relative weight of each criteria.

By calculating eigenvalues of the comparison matrix , the weight of each criteria can be obtained. It is determined by using following equations: $A. W = \lambda_{max} \cdot W$, where A represents the comparison matrix and λ_{max} the highest

eigenvalue. If there are n criteria to compare, the weight of each criteria can be stated as $W=(w_1, w_2, \ldots w_n)$. Ideally, in AHP the $a_{ij} = w_i/w_j$, however in realistic w_i/w_j usually unknown value.

Step 5. Determining the alternative

Finally, by using the weight coefficient value, the index of the rank alternative to obtain the goal can be achieved. To ease the implementation of AHP, the example below shows how one family wants to buy a new car.

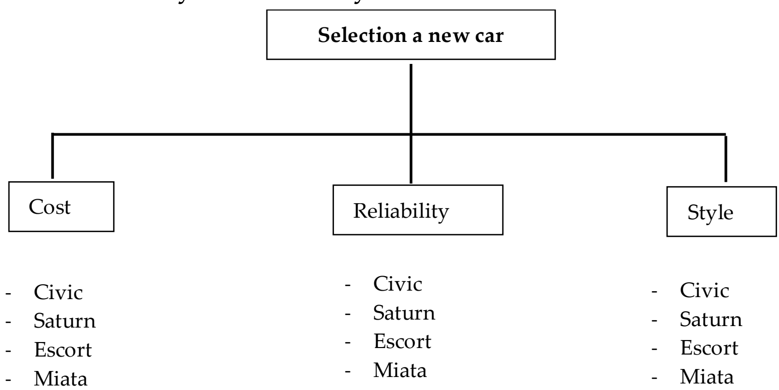

Figure 2.1 Sample illustration for selection a new car

There are 3 criteria to select a car : Cost, Style, and Reliability, and there are 4 alternative cars to choose : Civic, Saturn, Escort, and Miata. Family members can be represented as the decision makers. This situation can be depicted as shown on Fig 2.1.

	Cost	Reliability	Style
Cost	1	2	7
Reliability	$^1/_2$	1	5
Style	$^1/_7$	$^1/_5$	1

Figure 2.2 The comparison matrix for example on Fig 2.1

On Fig 2.1 , if one of member family has opinion that : cost is 2 times more important than reliability, cost is 7 times more important than style, and reliability is 5 times more important than style, then the comparison matrix can be stated as Fig. 2.2

Determining the weight for each criteria can be shown as Fig 2.3. The value of each column (criteria) is summed (Fig. 2.3(a)), and then is normalized (Fig.2.3(b)). As a last step, average of each row (criteria) is determined which can be defined as the weight of each criteria (Fig.2.3(c)). Based on result in Fig. 2.3, it can be concluded that the weight from one decision maker for cost, reliability and style are 0.59, 0.33, and 0.08, respectively.

$$\begin{array}{ccc} \begin{array}{c} \begin{bmatrix} 1 & 2 & 7 \\ ^1/_2 & 1 & 5 \\ ^1/_7 & ^1/_5 & 1 \end{bmatrix} \\ \begin{matrix} 1.64 & 3.2 & 13 \end{matrix} \\ \text{(a)} \end{array} & \begin{array}{c} \begin{bmatrix} 0.61 & 0.63 & 0.54 \\ 0.30 & 0.31 & 0.38 \\ 0.09 & 0.06 & 0.08 \end{bmatrix} \\ \\ \text{(b)} \end{array} & \begin{array}{c} \begin{bmatrix} \mathbf{0.59} \\ \mathbf{0.33} \\ \mathbf{0.08} \end{bmatrix} = \begin{bmatrix} \textbf{Cost} \\ \textbf{Reliability} \\ \textbf{Style} \end{bmatrix} \\ \\ \text{(c)} \end{array} \end{array}$$

Figure 2.3 Process determining the weight of each criteria.
(a) Comparison matrix, (b) Normalized, (c) Weight of each criteria

2.1.1. Fuzzy Preference Matrix

The fuzzy preference matrix (FPM) is first defined by Orlovski [4] to express the each pair of alternative a value that reflects the preference of the one alternative to the other one. One element in matrix A, a_{ij}, is defined that the preferred alternative i to j. On fuzzy preference relations, element comparison matrix is expressed into zero-to-one scale. It can be stated as a_{ij} which defines the dominance of alternative i over j, where $0<a_{ij}<1$, and $a_{ij}+a_{ji}=1$.

$$A=\begin{bmatrix} 0.5 & 1-a_{21} & 1-a_{31} & 1-a_{41} \\ a_{21} & 0.5 & 1-a_{32} & 1-a_{42} \\ a_{31} & a_{32} & 0.5 & 1-a_{43} \\ a_{41} & a_{42} & a_{43} & 0.5 \end{bmatrix}$$

Figure 2.4 Fuzzy preference matrix of AHP with 4 criteria

Fuzzy preference matrix A with 4 criteria can be depicted as Figure 2.4. There is one variant fuzzy preference matrix which is called incomplete fuzzy preference matrix [19-22, 29] It happens because a decision maker may develop a fuzzy preference relation with incomplete information. As Yejun et al. [19, 29] mentioned there are four causes :

(1) time pressure, lack of knowledge, and the DM's limited expertise related with problem domain;
(2) the number of the alternatives, n, is large. It makes it

is hard to accept all of the view of DM from all all the $n(n -1)/2$ required comparisons ;

(3) it can be convenient/necessary to skip some direct critical comparison between alternatives, even if the total number of alternatives is small ;

(4) an expert would not be able to efficiently express any kind of preference degree between two or more of the available options. This may be due to an expert not possessing a precise or sufficient level of knowledge of part of the problem, or because that expert is unable to discriminate the degree to which some options are better than others.

2.1.2. Multiplicative Preference Matrix

Saaty[2, 3] proposed the mutltiplicative preference matrix (MPM) which represents the ratio preference of one alternative compared to the other alternative. Element comparison matrix of multiplicative preference relation is composed according to one-to-nine point scale as illustrated in Table 2-1.

Table 2.1 Scale for multiplicative preference Matrix

Priority Scale	**Linguistic Meaning**
1	Equal important
3	Moderately more important
5	Strongly more important
7	Very strongly important
9	Extremely more important
2, 4, 6, 8	Intermediate values of important

Source: Saaty[2]

$$B = \begin{bmatrix} 1 & 1/b_{21} & 1/b_{31} & 1/b_{41} \\ b_{21} & 1 & 1/b_{32} & 1/b_{42} \\ b_{31} & b_{32} & 1 & 1/b_{43} \\ b_{41} & b_{42} & b_{43} & 1 \end{bmatrix}$$

Figure 2.5 Multiplicative preference matrix of AHP with 4 criteria

This element can be stated as b_{ij} which defines the dominance of alternative i over j, where $1 \leq b_{ij} \leq 9$, and $b_{ij} = 1/b_{ji}$. Multiplicative preference matrix B with 4 criteria can be depicted as Fig.2.5.

2.2. Consistent Ratio

One important issues in the AHP is the consistency of comparison matrix. Without satisfying the consistency, the comparison matrix as representative of decision maker(DM)'s opinion, cannot be used to make decision. If one DM said that he prefers apple than mango, and he/she prefers mango than watermelon, then it will be a logic opinion, if he prefers apple than watermelon. Moreover, it will be a illogic opinion, if he/she prefers watermelon than apple. This logic opinion can be said as the consistent relation, contrarily, it will be inconsistent if the opinion is

illogic. In small criteria, DM is easy to avoid from the inconsistent relation, but it is hard in large criteria.

Although AHP is effective for quantitatively judging the importance level of each criterion by its corresponding weight, there are some drawbacks of AHP which makes the inconsistent comparison matrix often occurs especially on the high number criteria. Triantaphyllou and Mann [30] argued that the traditional AHP proposes relative measurement of the various criteria is represented by set value discrete (1/9, 1/8, 1/7,…,7, 8, 9). In fact, some problems can be represented better on fuzzy set in continuous values than discrete value. They revealed this constraint causes the high failure rates. The study [31] found that it is hard to identify in-between members even it can easily to classify the representative members in a fuzzy set. As the consequence of the above weakness, the inconsistency of comparison matrix as behalf of the decision maker (DM) opinion often happens.

2.2.1. CR For Multiplicative Preference Matrix

Suppose a_{ij} is the element of a comparison matrix A which compares alternative i over j of a decision problem. The consistency of multiplicative preference matrix can be stated as two approaches. First, the a_{ij} are perfect consistent if they satisfy the following rules in Eq. 2.1.[10].

$a_{ij} = a_{ik}.\ a_{kj}$ where$i$, j, k, are criteria of comparison matrix **(2.1)**

The example of perfect consistent can be described as follows. Suppose you like an apple three times as much as mango (a_{ij}= 3) and a mango twice as much as a watermelon (a_{jk}=2). The perfect consistent is achieved when you like an apple six times as much as a watermelon (a_{ik}=6). It can be indicated as a consistent judgment if you said you like an apple five or seven (not six) times as much as a watermelon. However, it can be said as an inconsistent judgment if you said you like an apple than a watermelon. The second approach is determining the consistent ratio as the consistent degree of one comparison matrix. Saaty[2] gave the threshold of CR is 0.1. The value of CR is determined as shown Eq.(2.2), Eq.(2.3) and Eq.(2.4) :

$$A \times W = \lambda_{\max} \times W, \tag{2.2}$$

$$CI=(\lambda_{max} - n) / (n-1), \tag{2.3}$$

$$CR=CI / RI, \tag{2.4}$$

Where A is a comparison matrix, λ_{max} is the largest eigenvalue, W is the eigenvector of the matrix. Further, CI is consistent index, n represents number criteria or size matrix, and RI (random consistency index) is the average index of randomly generated weights. Value RI on each size matrices can be described on Table 2.2.

Table.2.2 Random consistency index (*RI*)

Number criteria	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
	0.58	0.9	1.12	1.24	1.32	1.41	1.45	1.49	1.51	1.48

Eigenvector W can be represented as a set of the weight of each criteria of matrix A, while λ can be represented as a set of scalar corresponding to eigenvector W as Eq. (2.2) [27]. Saaty's research conducted that if the more inconsistent opinion is mapped into matrix A, it makes λ_{max} of matrix A will be higher. Also λ_{max} will be higher than n (n= size matrix). So he made a formula to make a relation between λ_{max} and the rate of consistency as in Eq (2.3) and Eq. (2.4).

CI is influenced by the value of λ_{max}..If the matrix A is come from the perfect consistent judgment, λ_{max} *be* same with number criteria (= n). Also, if the matrix A is come from the consistent (not perfect) judgment, the different between λ_{max} and n is relative small. Therefore, the lower CI represents higher consistent (lower CR). To understand this concept, three types of judgment and their matrices are built as shown in Fig 2.6. The first type judgment is Allice's opinion. She said she likes an apple three times as much as mango (a_{12}= 3) and a mango twice as much as a watermelon (a_{23}=2). She also said he likes an apple six times as much as a watermelon (a_{13}=6). The second type judgment is Bob's opinion. He said he likes an apple three times as much as mango (a_{12} = 3) and a mango twice as much as a watermelon (a_{23}=2). He also said he likes a watermelon six times as much as an apple (a_{13}=1/6). The third type judgment is Charlie's opinion. He said he likes an apple three times as

much as mango (a_{12} = 3) and a mango twice as much as a watermelon (a_{23}=2). He also said he likes an apple nine times as much as a watermelon (a_{13}=9). Allice, Bob, and Charlie opinions are represented in matrices as shown in Fig 2.6 (a), Fig 2.6 (b), and Fig 2.6 (c), respectively. As a logic opinion, the first judgment can be indicated as the perfect consistent. It also can be proved by using Eq.(2.2), Eq.(2.3), Eq.(2.4). This judgment can be shown in Fig 2.6 (a). By using Eq.(2.2), λ_{max} is 3. Therefore, using Eq.(2.3), Eq.(2.4), CR will be 0 (perfect consistent).

$$\begin{bmatrix} 1 & 3 & 6 \\ 1/3 & 1 & 2 \\ 1/6 & 1/2 & 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & 3 & 1/6 \\ 1/3 & 1 & 2 \\ 6 & 1/2 & 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & 3 & 9 \\ 1/3 & 1 & 2 \\ 1/9 & 1/2 & 1 \end{bmatrix}$$

(a)	(b)	(c)
- CR = 0 (perfect consistent)	- CR = 1.383 (consistent)	- CR = 0.015 (inconsistent)
- λ_{max}= 3	- λ_{max}= 4.605	- λ_{max}= 3.017

Figure 2.6 Three types judgment in MPM (a) perfect consistent (b) consistent, (c)inconsistent

Contrarily, the second judgment is different with the first judgment. As a logic opinion, the second judgment can be indicated as the inconsistent opinion. By using Eq.(2.2), Eq.(2.3), Eq.(2.4), the CR will be 1.383 (>0.1). Although not perfect consistent, as a logic opinion, the third judgment is also indicated as consistent. By using Eq.(2.2), Eq.(2.3), Eq.(2.4), the CR will be 0.015 (< 0.1).

2.2.2. CR For Fuzzy Preference Matrix

Like multiplicative preference matrix, the consistency of fuzzy preference matrix is also exposed by two methods. First, E.Herrera-Viedma et al [32] reveals the "additive consistency" as the definition of consistency in a fuzzy preference matrix [33], which can be described on Eq. (2.5).

$$a_{ij} + a_{jk} + a_{ki} = \frac{3}{2}\ \forall i, j, k\ ; \qquad (2.5)$$

Where *i, j, k* is the criteria.

In the real, it can be said that if you prefer an apple to mango with value a_{ij}= 0.8 and prefer mango to watermelon with value a_{jk}= 0.6. The "perfect consistent" is achieved when you preferred apple to watermelon with value a_{ik}= 0.9 or a_{ki}=0.1. It can be determined as follows. The neutral is 0.5. Therefore if the comparison value is 0.8 and 0.6, the different preferred will be 0.3 (=0.8-0.5) and 0.1 (=0.6-0.5), respectively. So the total different will be 0.4 (=0.3+0.1). For above illustration, it is a logic value if the value a_{ik}=0.9 (=0.4+0.5).

Second, Xu and Da [34] determined the consistency in the fuzzy preference matrix by using Xu [35] method to reveal CI in multiplicative preference matrix. Suppose b_{ij} is the element of multiplicative of preference matrix. Xu [35] defined the CI in Eq. (2.6) which is derived from Eq. (2.2) and Eq. (2.3).

$$\boldsymbol{CI} = \frac{1}{n(n\text{-}1)}\sum_{1\le i<j\le n}\left[\boldsymbol{b_{ij}}\frac{w_j}{w_i} + \boldsymbol{b_{ji}}\frac{w_i}{w_j} - \mathbf{2}\right], \qquad \textbf{(2.6)}$$

Xu and Da [34] transformed the element of multiplicative preference matrix (b_{ij}) into the element fuzzy preference matrix (a_{ij}) by using $b_{ij}= a_{ij}/ a_{ji}$.. Therefore, CI is determined as shown Eq. (2.7).

$$\boldsymbol{CI} = \frac{1}{n(n\text{-}1)}\sum_{1\le i<j\le n}\left[\frac{a_{ij}}{a_{ji}}\frac{w_j}{w_i} + \frac{a_{ji}}{a_{ij}}\frac{w_i}{w_j} - \mathbf{2}\right], \qquad \textbf{(2.7)}$$

Finally, CR can be determined by used Saaty's formula CR = CI/RI.

To understand this concept, three types of judgment and their matrices are built as shown in Fig 2.7. The first type judgment is Allice's opinion. Allice's opinion is same with aforementioned in this sub section. Her opinion can be written a_{ij}= 0.8, a_{jk}= 0.6, and a_{ki}=0.1. This opinion can be categorized as a perfect consistent opinion as shown in Fig 2.7(a).

The second type judgment is Bob's opinion. His conclusion opinion can be written a_{ij}= 0.8, a_{jk}= 0.6, and a_{ki}=0.9. This opinion can be categorized as an inconsistent opinion as shown in Fig. 2.7(b).The third type judgment is Charlie's opinion. His conclusion opinion can be written a_{ij}= 0.8, a_{jk}= 0.6, and a_{ki}=0.3. This opinion can be categorized as a consistent opinion as shown in Fig. 2.7(c).The consistency of these opinions can be proved by using Eq. (2.5). However by using Eq. (2.7) and Eq. (2.4), the value CR is